

MATHEMATICAL READING LIST

A companion for future Mathematicians

"En guise d'Introduction"

This short selection of books is intended for freshmen in Mathematics looking for accessible complement to their education. They are all classical in their field; some are recommended in international renowned Universities, some are part of the personal library of professional mathematicians. Most of them are available in the UB library with freely downloadable digital edition (see https://www.ub.uni-bayreuth.de/).

The section "Readable Mathematics" provides a complement (not a replacement) to lecture notes that can appear dry at first, "Mathematical Philosophy" puts the mathematical thinking in a broader perspective (and could explain why some elementary ideas could feel familiar and elusive at once), "Panorama of Mathematics" proposes some large of focused views on topics of your studies that can seem disconnected and without goal nor motivations, "History of Mathematics" would be a reminder that no intellectual activity is disconnected from human life (for the best or for the worst). One could only regret the lack in this list of Russian, Chinese and Japanese representatives.

I hope that these works will bring to their reader as much mathematical appetite and enlightenment that they did for me and my colleagues. Most of them should be a motivation for active practice (take an ε , build a (counter) example, change the hypothesis, write a programme,...)

Viel Spass !! – B. Collas, Bayreuth, Germany.

READABLE MATHEMATICS

• WAS IST MATHEMATIK? (*What is Mathematics?*), R. Courant & H. Robbins, Springer-Verlag, xxii+400 pp., 2001. https://www.springer.com/de/book/9783540637776.

The two first years university mathematics syllabus (algebra, analysis, topology, number theory,...) with motivations and examples... and no compromise. Present key theorems and conjectures that motivated the development of the field. Best enjoyed with a friend in front of a blackboard. A must read.

• GÖDEL, ESCHER, BACH: EIN ENDLOSES GEFLOCHTENES BAND TASCHENBUCH (*Gödel, Escher, Bach: An Eternal Golden Braid*), D. R. Hofstadter, Deutscher Taschenbuch Verlag, 843 pp., 1992

Through the work of a logician, an artist and a composer, presents how the mathematical incompletness theorems of Gödel resonate at a cognitive level in terms of drawings and fugues. A fascinating piece of writing – "*A metaphorical fugue on minds and machines in the spirit of Lewis Carroll*".

For a short path to Gödel's theorems, see "Gödel's proof" by Nagel & Newman (150 pp.). The reader who is hooked by the Dedkind-Peano axioms and the question of foundations could look further to Zarmelo-Fraenkel & Choices, Tarski–Grothendieck theory and their implications for undergraduate mathematics.

• EINE EINLADUNG IN DIE MATHEMATIK: EINBLICKE IN AKTUELLE FORSCHUNG (*Invitation to Mathematics*), D. Schleicher & M. Lackmann Ed., Springer-Verlag, xv+228 pp., 2013. [+ Ebook]

A collection of short talks by researchers presented to the participants of the International Mathematical Olympiad, the goal being to build a bridge between studies and research. *"Research mathematics is like wildlife in uncharted territory, whereas olympiad problems are like animals in a zoo* [...]" – Jószef Pelikán.

MATHEMATICAL PHILOSOPHY

• THINKING ABOUT MATHEMATICS: THE PHILOSOPHY OF MATHEMATICS, S. Shapiro, Oxford University Press, 328 pp., 2000

Presents the mathematical questions that have motivated philosophy from Plato to Kant and reaches the famous opposition Logicism vs Formalism vs Intuitionism. Bring the novice reader up to date to the contemporary philosophy of mathematics. The chapter on Kant is renowned on its own.

• WEGE DER WISSENSCHAFT: EINFÜHRUNG IN DIE WISSENSCHAFTSTHEORIE (*What is this thing called Science?*), A. F. Chalmers, Springer-Verlag Berlin Heidelberg, 2001. [+ Ebook]

To reduce sciences to Gallileo's "the Book of Nature is written in mathematical language" would forget 400 years of epistemology developments. Chalmers provides an accessible account of modern philosophers (Popper, Kuhn, Feyerabend) on what are sciences, the role of hypothesis, and their connection to the world and human society. A classical for every scientists.

• WISSENSCHAFT UND HYPOTHESE (*La Science et l'hypothèse*), H. Poincaré, Xenomoi Verlag, 360 pp., 2003. [+ Ebook]

Written by the last universal mathematician in 1902. Explore the role of mathematics, hypothesis, abstract reasoning and experiments in physics, biology and geometry. A confrontation for the reader to the statement: "To the superficial observer scientific truth is unassailable, the logic of science is infallible; and if scientific men sometimes make mistakes, it is because they have not understood the rules of the game [...] This, to the minds of most people, and to students who are getting their first ideas of physics, is the origin of certainty in science." – H. Poincaré.

• MATHEMATICS WITHOUT APOLOGIES: PORTRAIT OF A PROBLEMATIC VOCATION, M. Harris, 464 pp., 2015.

Referring to Hardy's "A Mathematician's Apology", a former Professor in Paris, now in Columbia University, provides an overview of the intellectual and ethical challenges meet by mathematicians from the XXIth century (think A.I., algorithms and informations, or independent thinking of the citizen). A discussion based on classical and popular culture.

"A science is said to be useful if its development tends to accentuate the existing inequalities in the distribution of wealth, or more directly promotes the destruction of human life." - G. H. Hardy (1940).

PANORAMAS OF MATHEMATICS

• A PANORAMA OF PURE MATHEMATICS (AS SEEN BY N. BOURBAKI) (*Panorama des mathématiques pures. Le choix bourbachique*), J. Dieudonné, Academic Press Inc, 289 pp., 1982; [+ Ebook]

Written by a French former member of the Bourbaki secret mathematical circle which is at the origin of the international reform of mathematics in the 30s. An overview of pure mathematics that gives an idea of what graduate mathematics look like. Better suited to third years students.

• THE PRINCETON COMPANION TO MATHEMATICS, T. Gowers & al., Princeton University Press, 1034 pp., 2008. [+ Ebook]

Under the direction of Tim Gowers Field medallist, 150 international mathematicians present 26 "Branches of Mathematics" (e.g. arithmetic geometry, dynamic, stochastic process), 99 concepts (e.g. logarithm, dimensions) as well as "The Influence of Mathematics" and "What is Mathematics?" Not comprehensive hence not an encyclopedia but an enjoyable companion for your studies that will grow with your discovery of the field.

• FINDING MOONSHINE: A MATHEMATICIAN'S JOURNEY THROUGH SYMMETRY, M. du Sautoy, Fourth Estate, 375 pp., 2008

An Oxford professor bring us through the development of the concept of symmetries (aka group theory) in geometry and equations: from its first appearance in France with E. Galois to the international competition in obtaining a complete classification – started in 1965, finished in 1980, still being checked today. Moonshine refers to a surprising connection between the Monster symmetry group ($\sim 8.10^{53}$ elements) and a fundamental function in number theory. Another acclaimed publication of the author is "The music of Primes" on the Riemann Hypothesis.

• FERMATS LETZTER SATZ: DIE ABENTEUERLICHE GESCHICHTE EINES MATHEMATISCHEN RÄT-SELS (*Fermat's Last Theorem*), S. Singh, dtv, 368 pp., 2000.

A must read popular mathematical book. On a generalization of Pythagorean triplets which started as a claim in the margin of a book in Toulouse (1637) and lead the development of number theory until A. Wiles' proof (Oxford, 1995).

HISTORY OF MATHEMATICS

• MEN OF MATHEMATICS, E. T. Bell, Touchstone pub., 608pp., 1986.

An history of men and ideas in sciences, from the fifth century BC with Zeno to 1918 with Kantor. Follow the development of mathematics concepts (symmetries, infinity, groups, arithmetic, logic,...) through men – "*The science of Pure Mathematics, in its modern developments, may claim to be the most original creation of the human spirit.*" – A. N. WHITEHEAD.

• EMMY NOETHER, DIE NOETHER-SCHULE UND DIE MODERNE ALGEBRA, M. Koreuber, Mathematik im Kontext, Springer Verlag, xv+368, pp., 2015. [+ Ebook]

From Erlangen to Göttigen then Princeton, the role of E. Nother (and of the "Noether's boys") in the development of abstract formal commutative algebra as we know it today, as well as in Galois theory, physics, and topology. Mathematical progress in parallel to social one when professorship in Germany was still forbidden to women.

• A RUSSIAN CHILDHOOD, S. Kovalesvskaya, Springer Verlag, xiv+262 pp., 1978. Available at https://www.springer.com/de/book/9780387903484 [Ebook].

Sofia Kovalevskaya was the first woman to be appointed professor in Europe (Stockholm 1883). Some incredible memories on her political life (nihilist in Russia, socialist in Germany, witness of the "Commune de Paris") and mathematical achievements (Weierstrass analysis, Cauchy differential equations.) An alternative is M. Audin's "Remembering Sofya Kovalevskaya" [+ Ebook].

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"En guise de Conclusion"

- (i) Can we build a geometric continuous surjection from [0, 1] to $[0, 1]^2$?
- (ii) What is the dimension of the Koch curve?
- (iii) Is there a Fermat Theorem for polynomials?
- (iv) Is the sum of the angles of a triangle always equal to 180 degrees?