UNIVERSITÄT BAYREUTH

MATHEMATISCHES INSTITUT
LEHRSTUHL IV -ZAHLENTHEORIE

# GEOMETRY: FROM SPACES TO ALGEBRAIC STACK

Benjamin Collas

Advanced Lecture - SS 2018

## Description

This lecture deals with the notion of geometry following A. Grothendieck's homotopy categorical approach in terms of sheaves on a category with respect to a given topology. Starting with the example of topological varieties, then the category of schemes, the goal of this lecture is to show how this approach gives access to various classical geometrical contexts in algebraic geometry that finally leads to the category of algebraic stacks.

The notion of stacks can be seen as an enhancement of the notion of schemes by allowing to deal with "bad" group action on spaces, and by providing fine answers to moduli classification problems. Typical examples encountered in algebraic geometry are given by moduli spaces of curves, Hurwitz spaces of covers or vectors bundles.

While some familiarity with algebraic geometry and schemes is assumed – e.g. as given in *Algebraische Geometrie (Prof. Dr I. Bauer WS 17/18)* – the required reminders will be given following the background of the participants. This advanced lecture is aimed at students willing to broaden their knowledge on the last recent advances in homotopical algebraic geometry.

## Topological Varieties

We introduce the functorial notions of this lecture through the example of the category of Topological Varieties that we recover as a full subcategory of sheaves on a Grothendieck site. We characterise this category and build the notion of geometrical context that we develop in the rest of the lecture.

*Keywords: Yoneda embedding, representable, Grothendieck site, P-locality, example of quotient varieties.*

## Schemes and Algebraic Spaces

We define the geometrical-algebraic context modelled on the category of affine schemes that recovers the category of schemes and extends to the notion of algebraic spaces. We give geometrical examples as well as counter-examples to representable.

*Keywords: flat, fpqc, étale morphisms; affine, projectives schemes; Zariski space, étale topos.*

## Algebraic Stacks & Homotopy Theory

We present the notion of stacks, then algebraic and Deligne-Mumford stacks in terms of the homotopy category of sheaves on the Grothendieck site of affine schemes and within the geometrical-algebraic context previously built. We introduce their main companions that are the stack inertia and homotopy group sheaves, as well as examples from geometry as solutions to a fine moduli classification problem.

*Keywords: homotopic theory, groupoids, local equivalence, stack inertia, quotient stack, moduli stack of curves.*

Please consult Elearning for schedule, rooms and references, and the lecturer if interested.